



Bispectrum and Bicoherence In Dipole Confined Plasmas

B.A. Grierson
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Outline

- Producing high density plasmas.
- General plasma characteristics.
- Fluctuation properties.
- Bispectrum.



CTX Dipole

$B_{\max} \sim 2\text{kG}$, $B_{\text{wall}} \sim 50\text{G}$

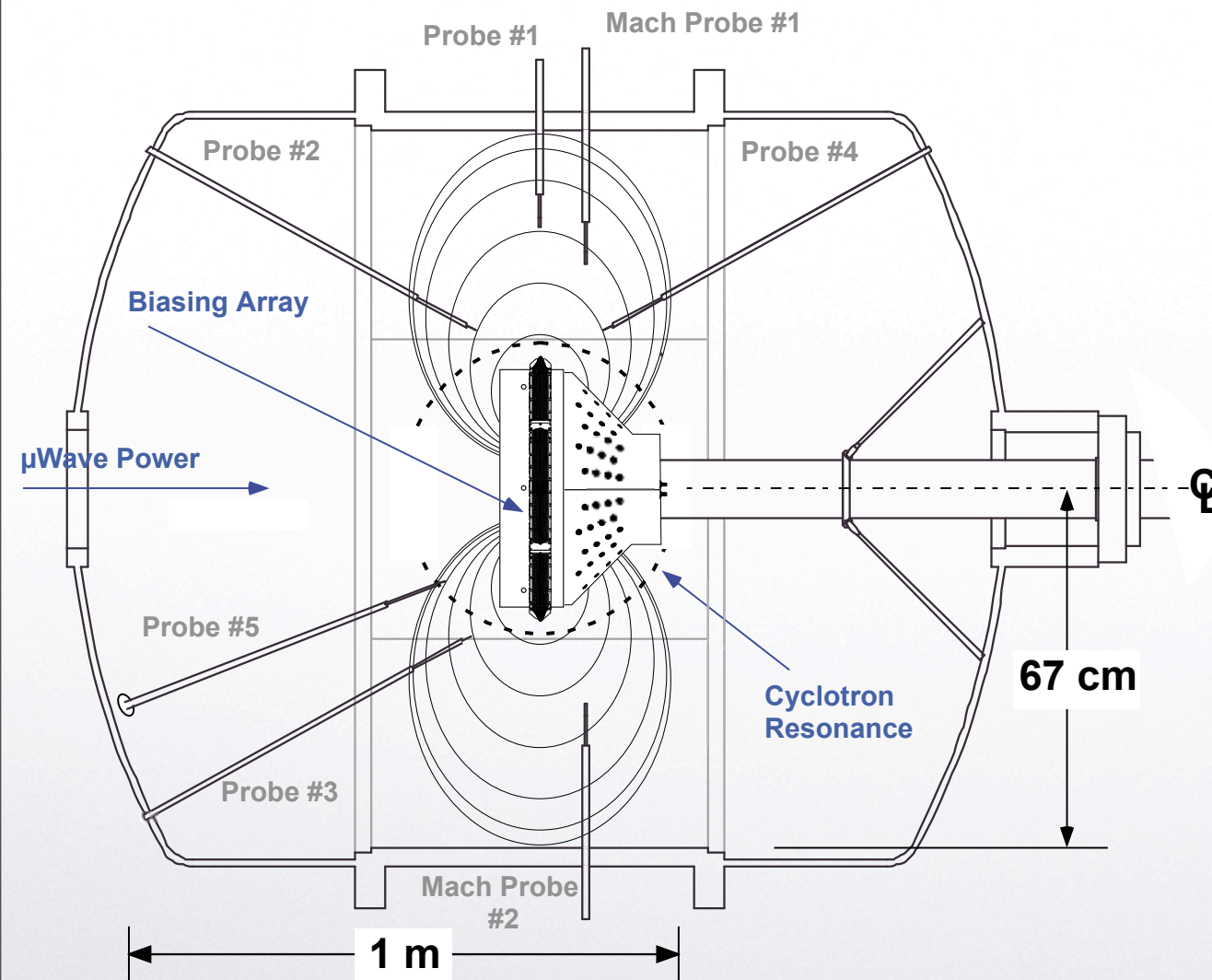
$L_{\text{Terella}} = 20\text{cm}$

$L_{\text{Chamber}} = 70\text{cm}$

1kW ECRH @2.45GHz

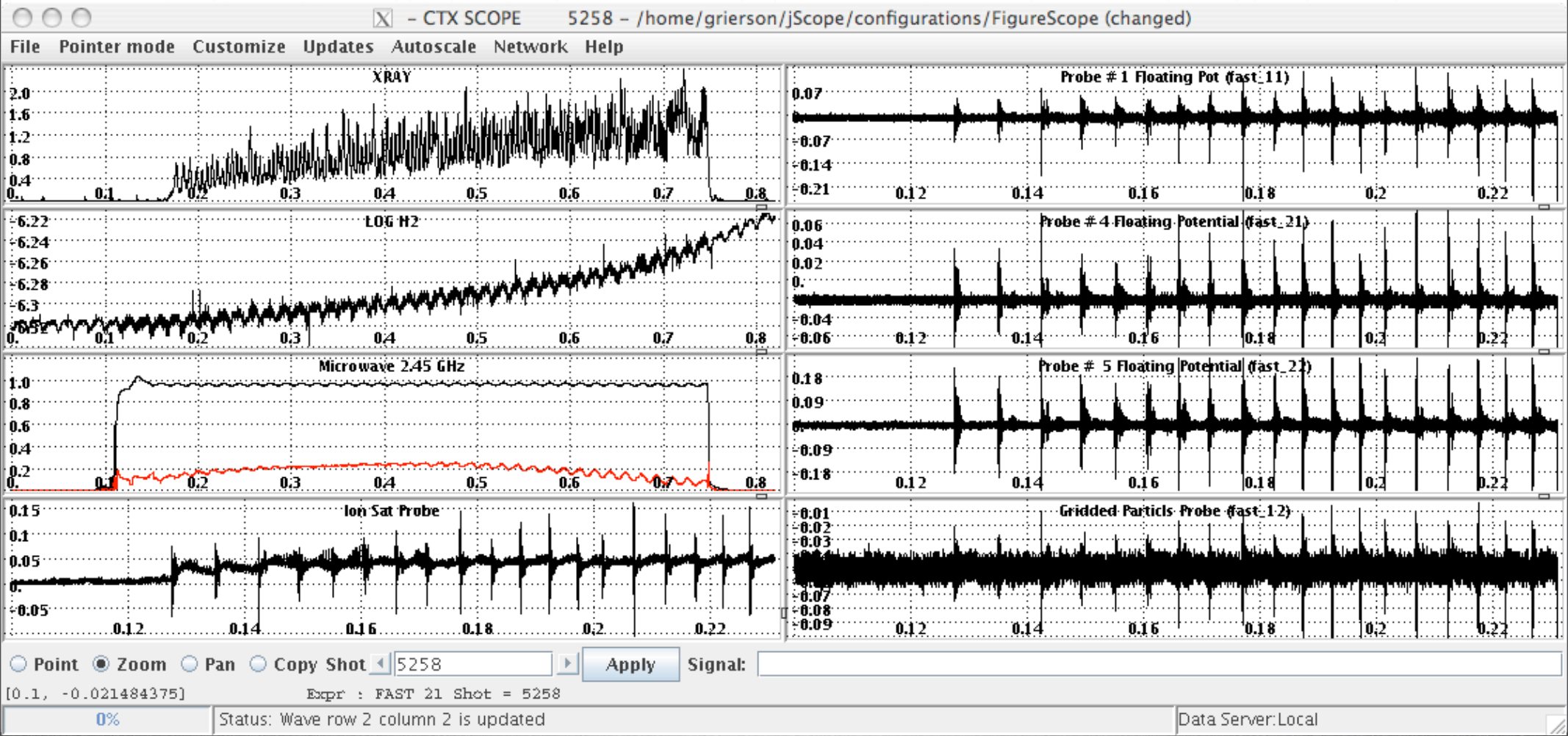
ECRH Resonance at

$L=27\text{cm}$



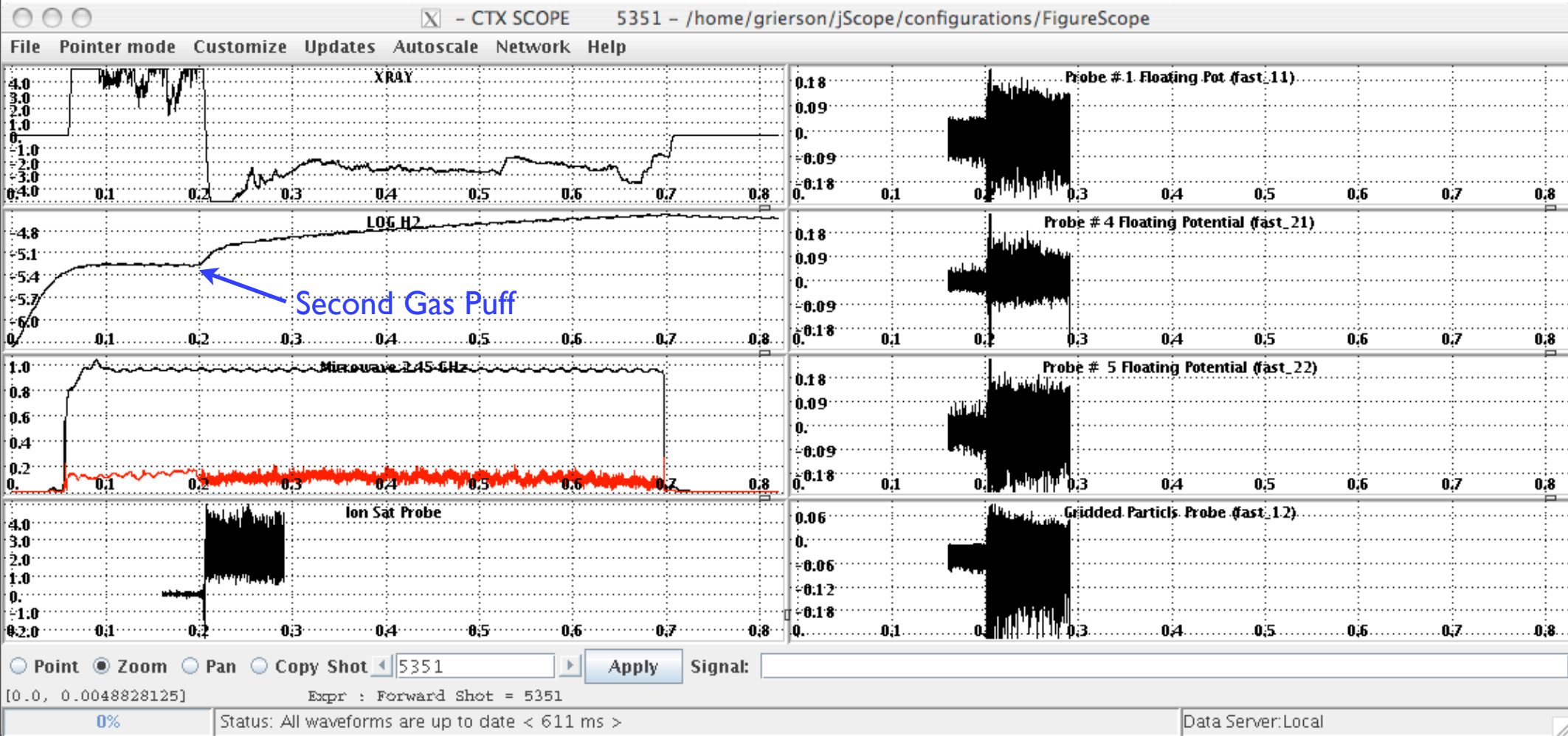


Low Density Plasma



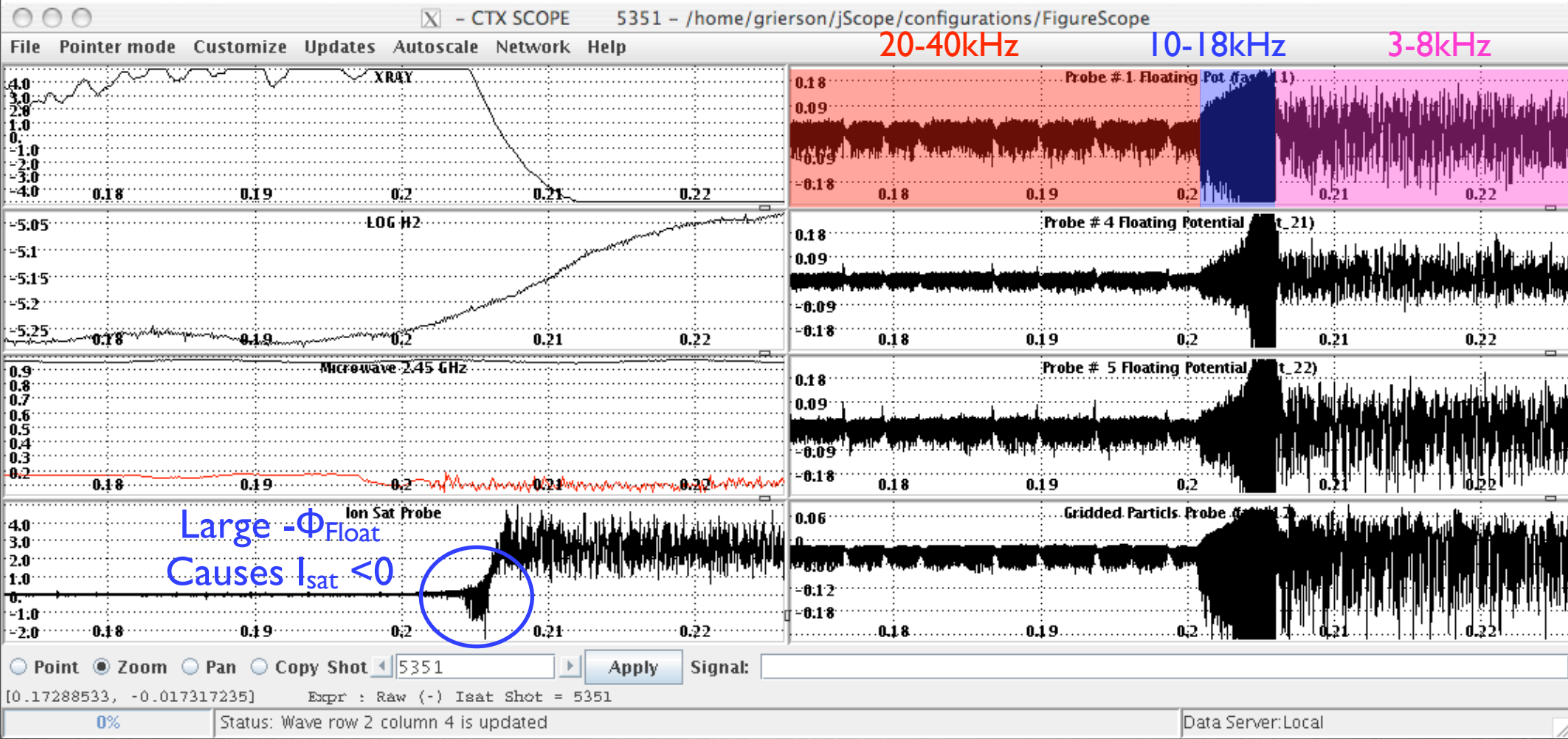


High Density Plasma



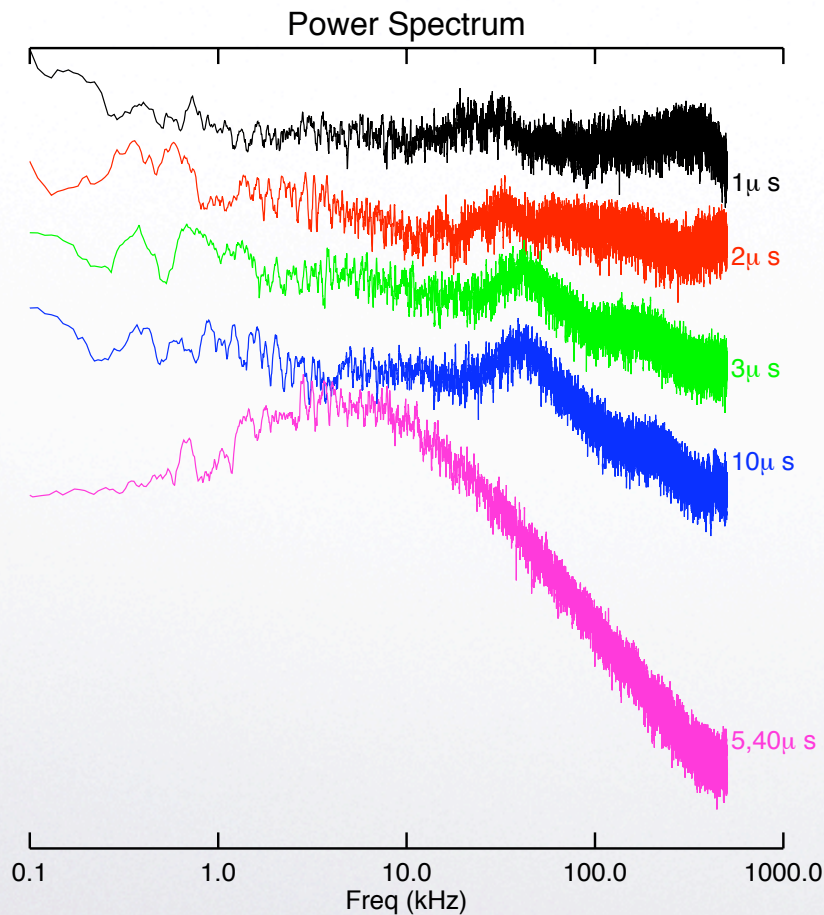


Transition





Spectral Characteristics



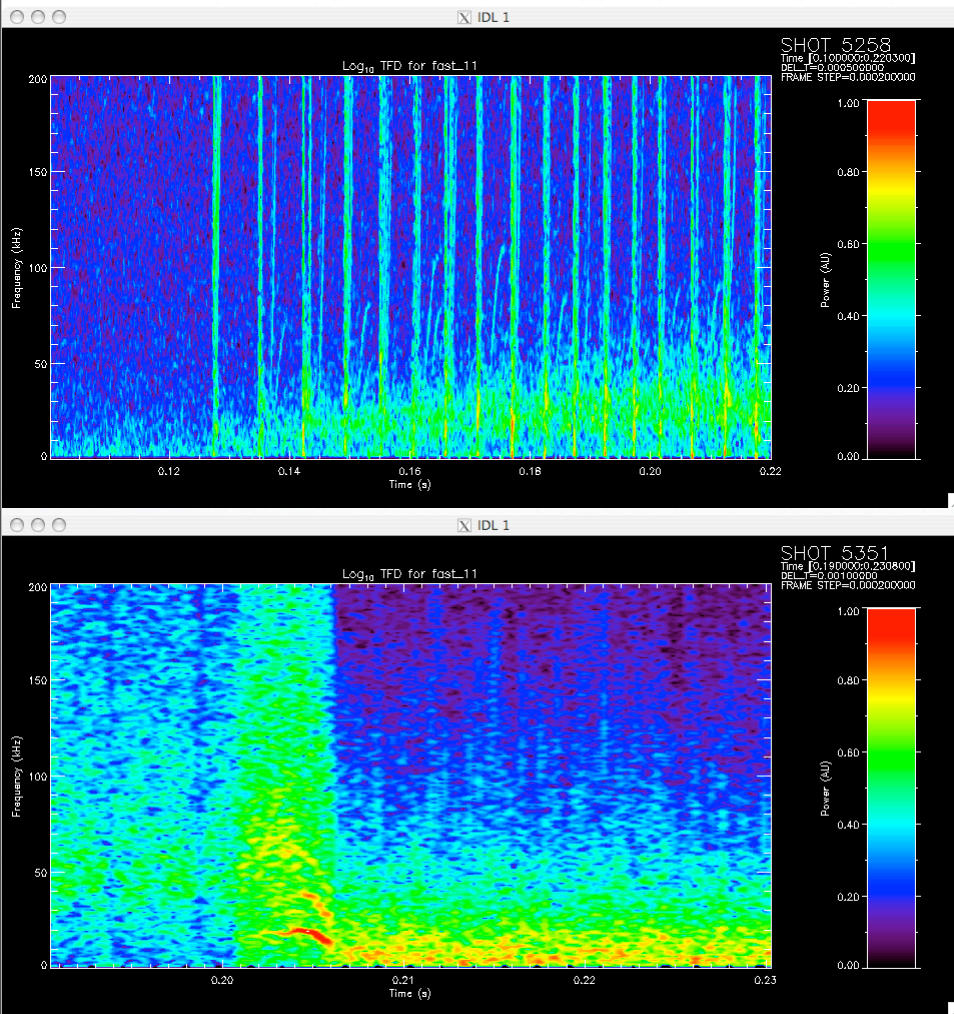
Floating Potential FFTs include HEI for low density.

- Trend towards power law
- Low density fluctuation peak shifts up as gas increased
- High density power law like $f^{-5/3}$ (which possibly corresponds to k-space as well)*

*S.-I. Itoh and K. Itoh **Spectrum of Subcritically Excited Interchange Mode Turbulence**



TFDs



- **Low density** plasmas have **turbulent** interchange fluctuations near 20-40kHz, with **little to no measurable mode structure**.
- **Transition** fluctuations near 10-18kHz and **m=1** mode structure.
- **High density** plasmas have fluctuations near 3-8kHz, dominated by **m=1**.



Correlations

- Correlations between probes can replace FFT phase measurements for non-stationary time series.
- Allows extraction of lag time.
- Lag time + <Frequency> \Rightarrow <Phase Shift>

$$C_{1,2}(\tau) = \frac{\int_0^T S_1(t)S_2(t - \tau)dt}{\sqrt{\int_0^T S_1^2(t)dt \int_0^T S_2^2(t)dt}}$$

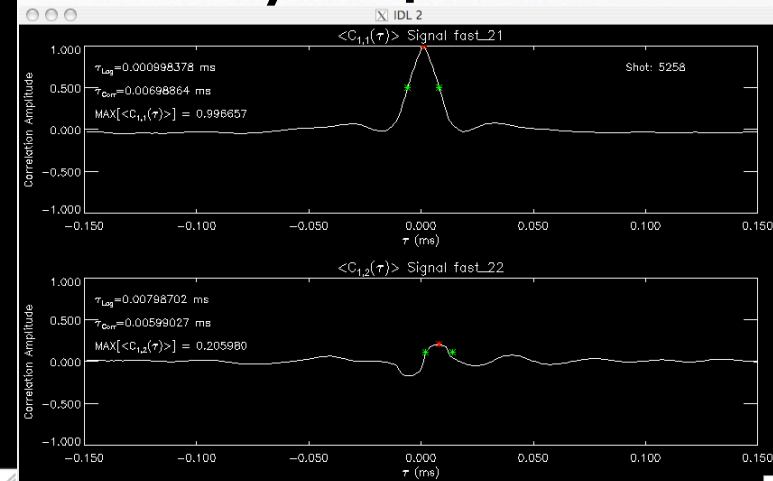
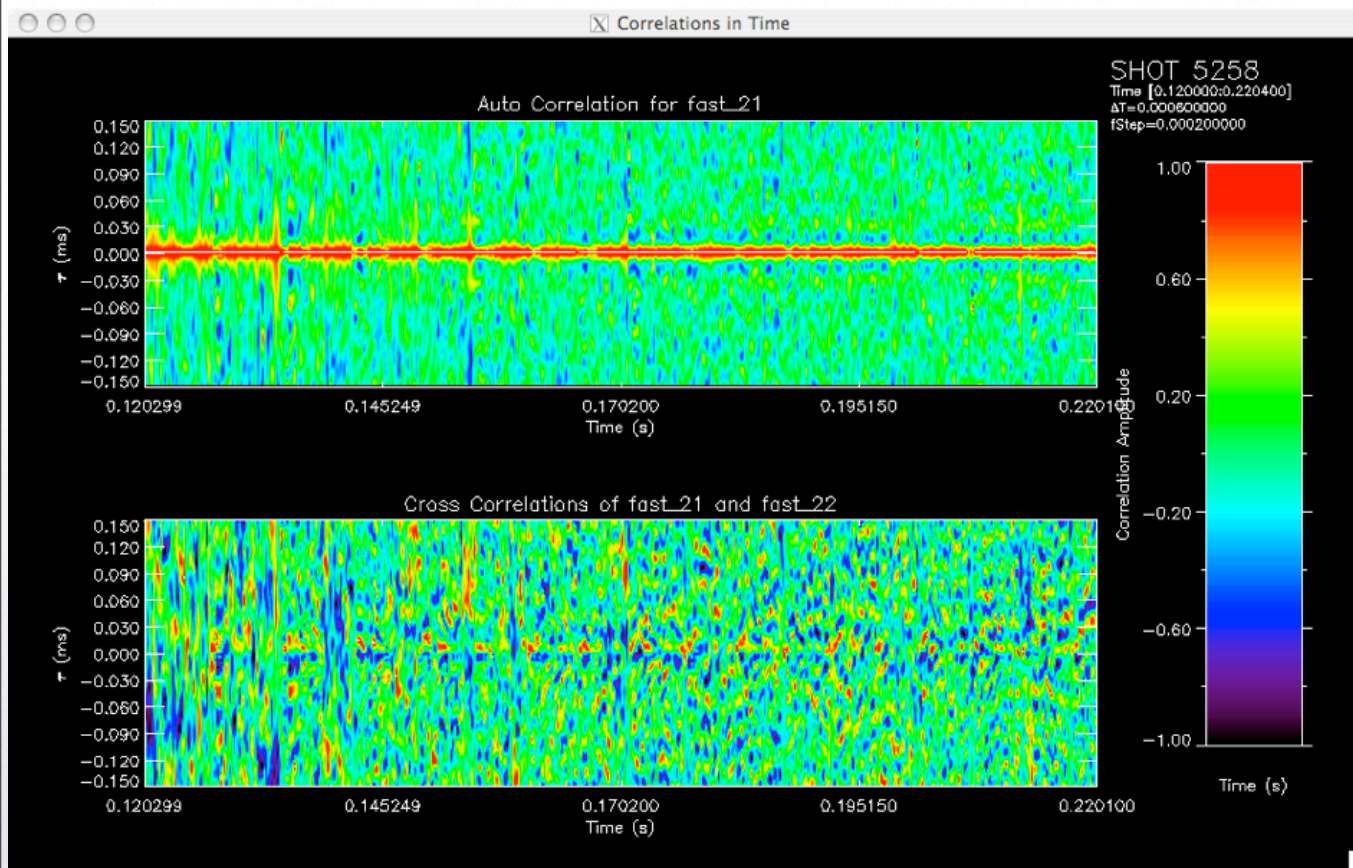
$$C_{1,2}(t, \tau) = [C_{1,2}^{(1)}(\tau), C_{1,2}^{(2)}(\tau), \dots, C_{1,2}^{(M)}(\tau)]$$

$$\langle C_{1,2}(\tau) \rangle = \frac{1}{M} \sum_{i=1}^M C_{1,2}^{(i)}(\tau)$$



Low Density $C_{1,2}(t, \tau)$

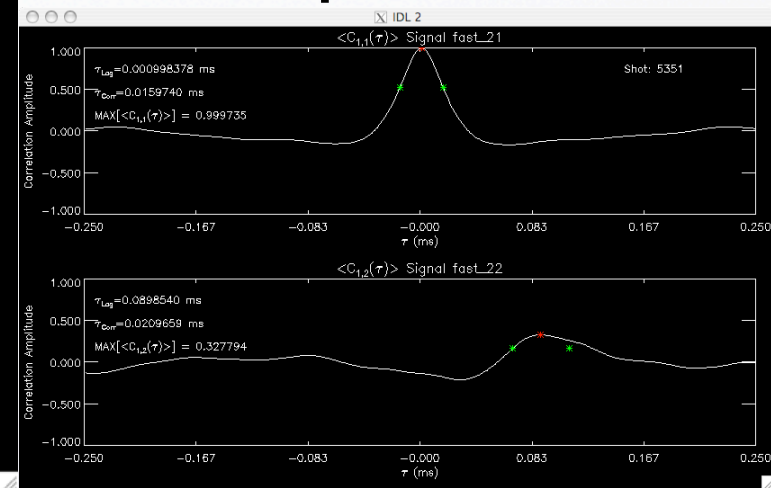
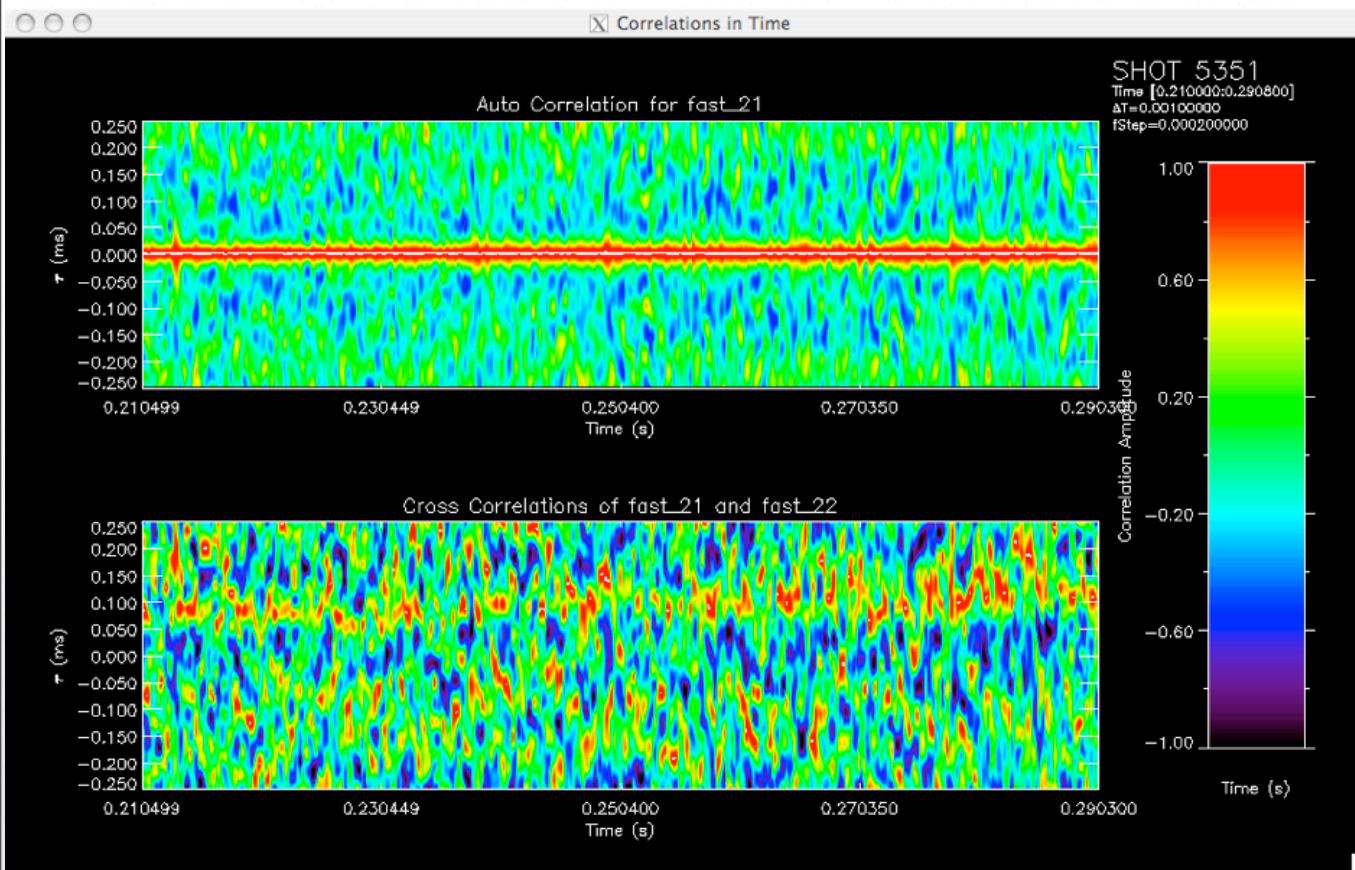
$\sim 8\mu\text{s}$ Lag and
30kHz fluctuations
give $\alpha = 86^\circ$ for
probes separated
by $\Delta\varphi = 90^\circ$.





High Density $C_{1,2}(t, \tau)$

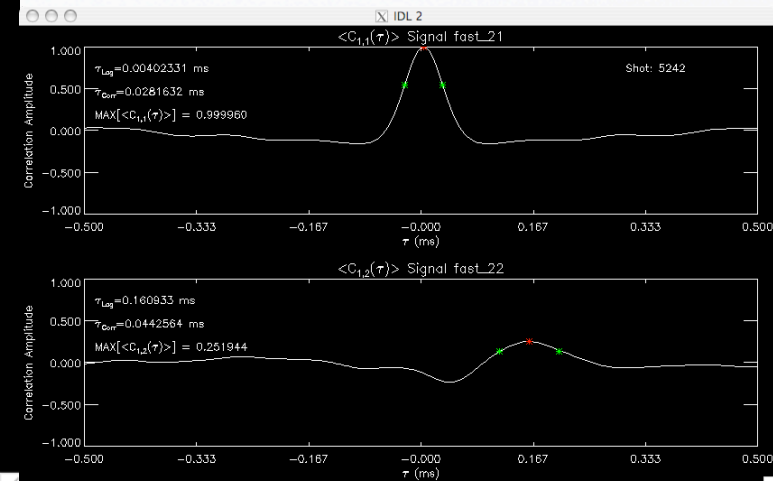
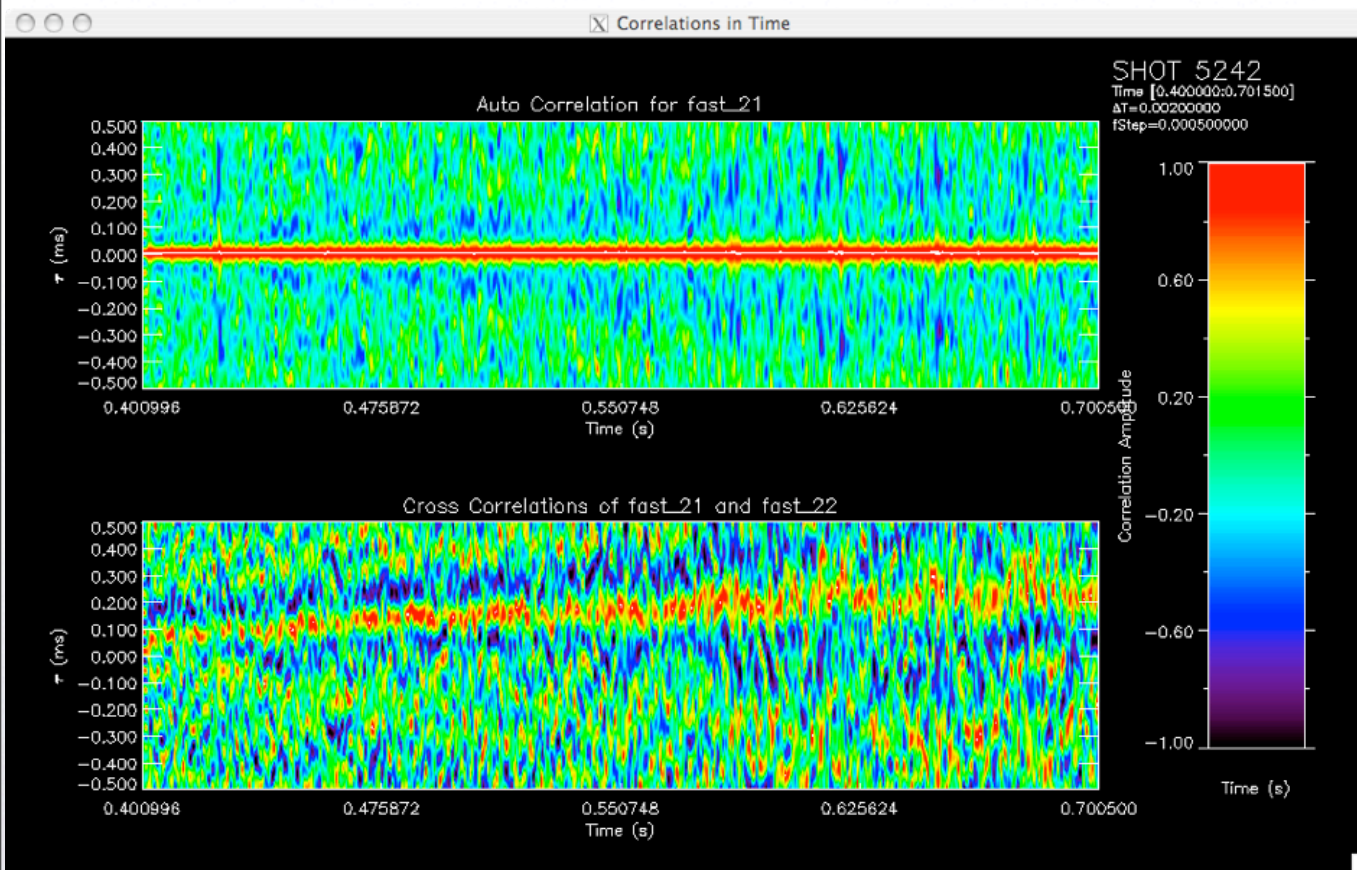
~89 μ s Lag and
3-4kHz fluctuations
give $\alpha=90-110^\circ$ for
probes separated by
 $\Delta\varphi=90^\circ$.





Longer Time High Density

As the discharge evolves, $\tau_{\text{Lag}} \uparrow$
and $f \downarrow$, keeping an
 $m=1$ mode structure.





Bicoherence

- Transform a time series to the frequency domain.
- Create the Bispectrum over many records (ensemble average)
- Form power-weighted Bispectrum (bicoherence) after M samples have been taken.
- 95% confidence for $b^2 > 3/M$ *

$$S(t) \xrightarrow{FFT} \hat{S}(\omega)$$

$$\langle A \rangle = \frac{1}{M} \sum_{i=1}^M A_i$$

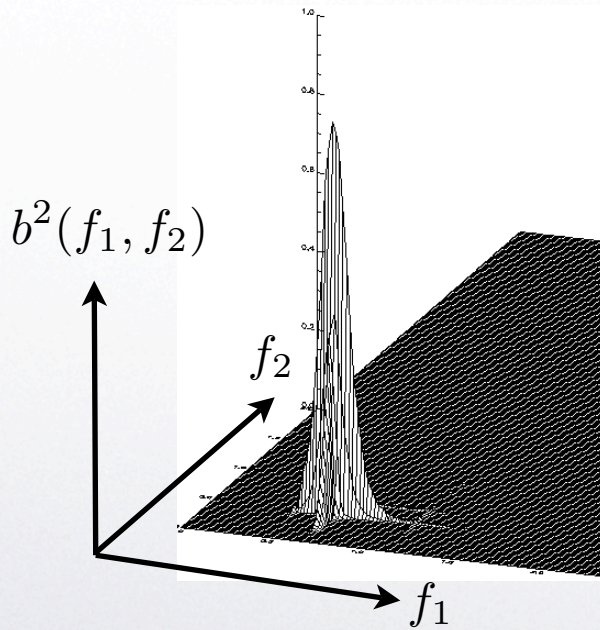
$$\hat{B}(\omega_1, \omega_2) = \langle \hat{S}(\omega_1) \hat{S}(\omega_2) \hat{S}^*(\omega_1 + \omega_2) \rangle$$

$$\hat{b}^2(\omega_1, \omega_2) = \frac{|\hat{B}(\omega_1, \omega_2)|^2}{|\langle \hat{S}(\omega_1) \hat{S}(\omega_2) \rangle|^2 |\langle \hat{S}(\omega_1 + \omega_2) \rangle|^2}$$

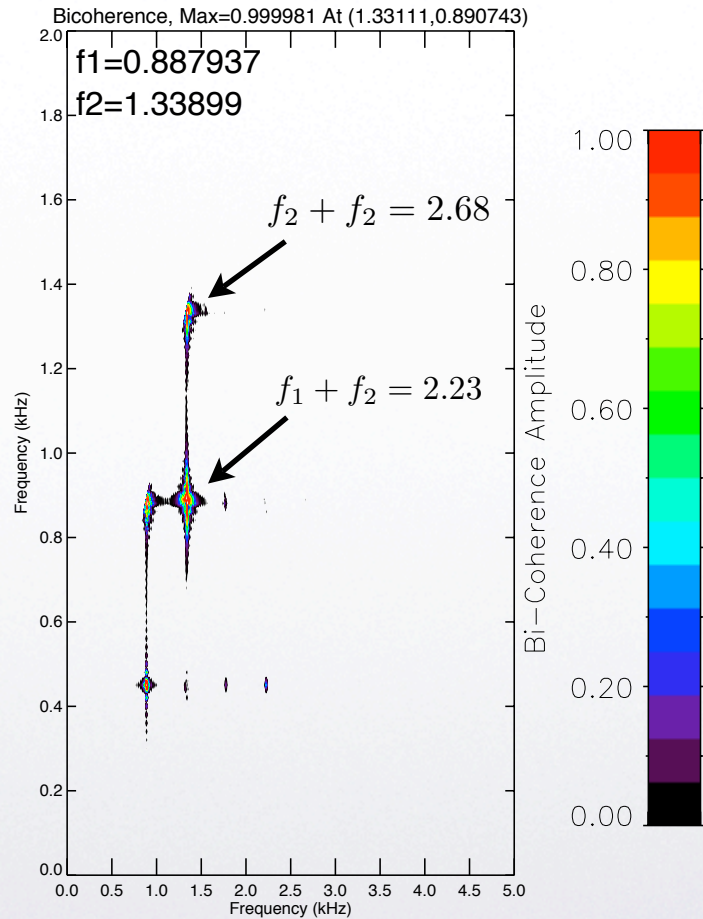
*V.Nosenko, J.Goree, and F.Skiff, Phys. Rev. E 73 016401 (2006)



Bicoherence Surface



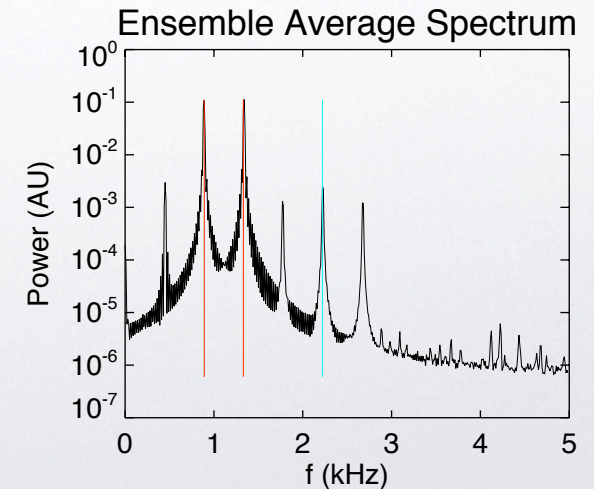
Examples



$$S_L(t) = \sum_{i=1}^2 \sin(2\pi f_i t)$$

$$S_{NL}(t; \epsilon) = S_L(t) + \epsilon S_L(t)^2$$

Strongest 3-wave coupling at sum $f_1 + f_2 \sim 2.23$, although others exist.





Example Biphas and Amplitude

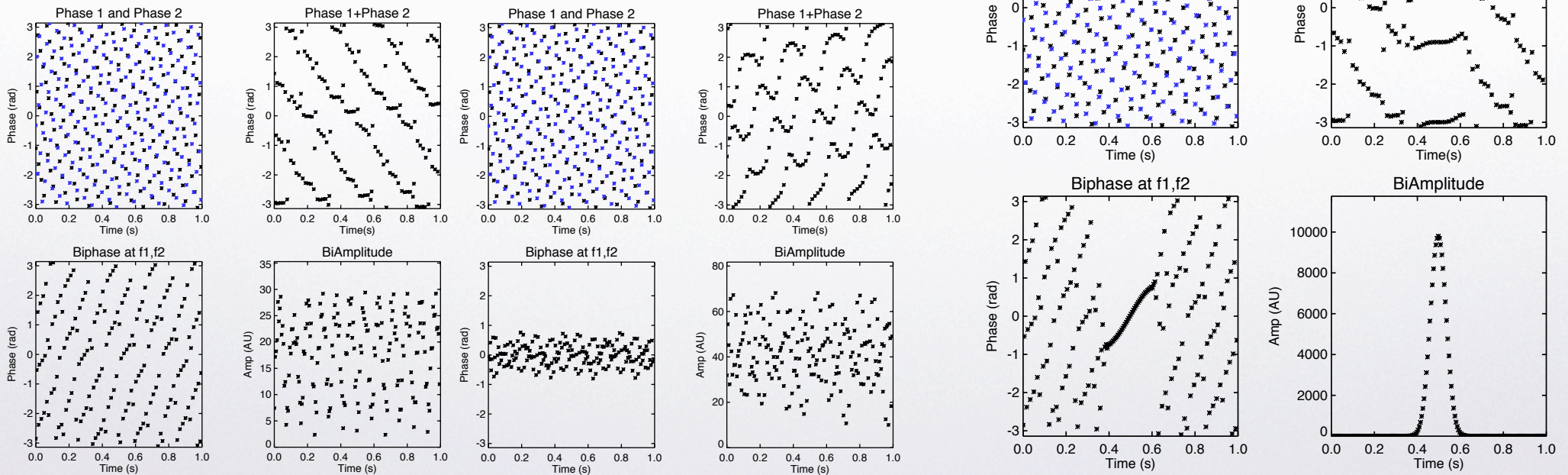
$$B(\omega_1, \omega_2) = |B(\omega_1, \omega_2)| e^{i\angle B(\omega_1, \omega_2)} = A e^{i\varphi_B}$$

$$S_{NL}(t : \epsilon) = S_L(t) + \epsilon S_L(t)^2$$

$\epsilon=0.0$

$\epsilon=.1\%$

ϵ is a Gaussian [0, 1]





The Analysis Procedure

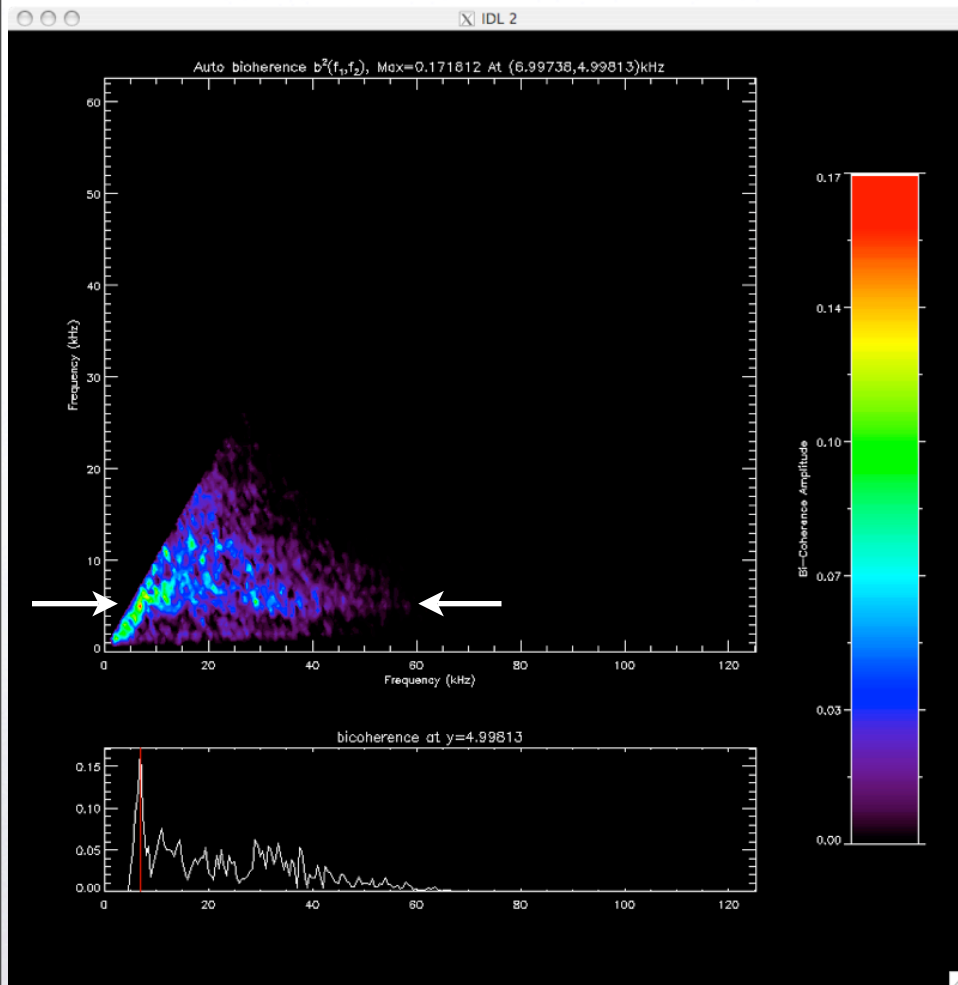
- For the following figures, 740 records have been taken to calculate the bicoherence.
- The records overlap by 75% to accurately measure the biphase evolution.
- The frequency pair where the Max Bispectrum occurs is tracked in time, as well as the amplitude (BiAmplitude).
- The BiAmplitude is qualitative, and measures the **intensity** of mode-mode coupling **in time**.
- The frequency pairs record **where** (in frequency-space) the coupling occurs **in time**.

This rather lengthy explanation is necessary, because we are using a Fourier Mode technique to measure non-stationary fluctuations. Dominant frequencies evolve, making the **bicoherence** a 'smeared-out' statistical measure.



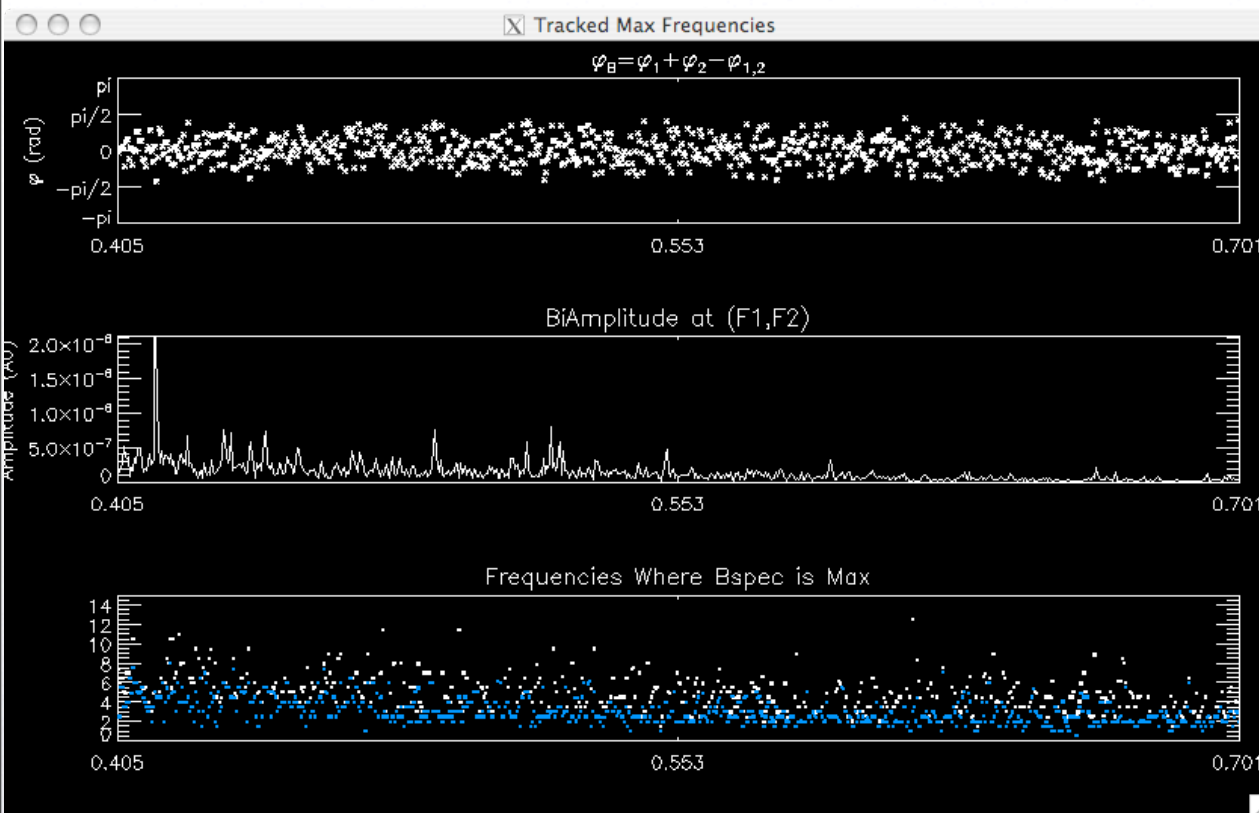
Bicoherence

- Bicoherence Max at $(f_1, f_2) = (7, 5)$ kHz indicating mode-mode coupling (not harmonic).
- Coupling exists above statistical cutoff (0.004) across many frequency pairs (triangle-like region).
- 5kHz mode coupled to 7-40kHz modes (white arrows).





Biphase



- Tracked pairs maintain a biphase close to zero (phase coupled)
- BiAmplitude displays intermittency, but decreases over time.
- Frequencies at Max BiSpectrum decrease.



Summary

- High density plasmas have been formed in the CTX device by increased fueling.
- The increased fueling causes a suppression of the Hot Electron Interchange mode, but gives rise to low frequency (3-8kHz) turbulent interchange modes.
- Low frequency modes have $k_{||} \approx 0$ (not shown) and power-law frequency spectrum.
- Bicoherence for mode-mode coupling significant (Dominated by non-linear processes)
- Frequency of fluctuations decrease as neutral pressure continues to rise.